


Sequential Feedback-Based Phase Optimization Using Hadamard Basis for Wireless Power Transfer

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Abstract—Wireless Power Transfer (WPT) is a promising technology with the potential to enable efficient energy delivery to devices at long distances. A major challenge lies in searching the optimized phase pattern of transmitter arrays (TXAs) due to the various kinds of possible phase combinations. This paper introduces a low-complexity algorithm that employs the orthogonal basis of Hadamard matrices to streamline the search for optimal phase distributions. By leveraging the orthogonality of Hadamard matrices, the proposed method achieves efficient and iterative phase optimization with minimal hardware requirements. The system focuses only on phase adjustments to maximize received RF power at the receiver array, making it practical for real-world implementations. Simulation and experimental results validate the effectiveness of the proposed approach, demonstrating its feasibility and simplicity.

Index Terms—Beam-scanning, Microwave Power Transmission (WPT), Power Transfer Efficiency (PTE), Wireless Power Transfer (WPT), Wireless Power Transmission (WPT).

I. INTRODUCTION

WIRELESS Power Transfer is a promising technology that can enable a truly wireless world [1]. It comprises three major types: the inductive coupling method, which operates at very short distances and has been successfully commercialized as the Qi standard [2]; resonant coupling; and microwave transmission. These technologies continue to advance, paving the way for future industrial adoption [3]. Among these, the use of electromagnetic waves is the only method capable of transmitting power over long distances by sending RF signals directly. By utilizing phased array antennas as transmitters, Wireless Power Transfer via microwaves (hereafter referred simply to as WPT) adjusts the phase and magnitude to control the beam direction and optimize efficiency for the receiver, which can be any target device in need of charging.

However, controlling both the magnitude and phase to focus the beam on the receiver in real-time is highly challenging. Since the phase primarily determines the beam direction and the magnitude affects efficiency¹, even deciding the phase alone is complex. This complexity increases significantly with

¹In WPT scenarios, efficiency is commonly referred to as Power Transmission Efficiency (PTE), which is defined as the ratio of the total received RF power to the total transmitted RF power.

larger arrays. Therefore, it is crucial to determine an efficient phase distribution that delivers high power to the receiver in a very short time. Various algorithms have been proposed to rapidly determine the optimal phase and magnitude distribution, including time-reversal [4], [5], backscattering [6], retro-directive [6], and feedback-based methods [7]. Each algorithm has its strengths and weaknesses in terms of hardware complexity, algorithmic complexity, and optimality.

Among these, the feedback-based algorithm [7], [8] has its advantages due to the simplest hardware structure, as it only requires a simple feedback module from the receiver while the transmitter only requires a simplest yet essential power path. This is a well-known methodology in telecommunications, commonly used for improving Channel State Information (CSI) and similar tasks. However, implementing this methodology in the WPT field is challenging because of the limited information available compared to the vast possibilities of magnitude and phase configurations in the phased array antenna (transmitter array). The system relies solely on feedback information, such as the received RF or DC power at the receiver, to estimate the optimal pattern. Since each feedback iteration takes time, it is crucial to make efficient decisions during every iteration.

In summary, this paper proposes a feedback-based algorithm inspired by [7], which utilized Hadamard matrices as a basis to sweep phases. Building upon this prior work, the proposed method enhances the technique by incorporating a sequential cumulative approach and synthesize the 2D pattern by two 1D patterns. The effectiveness of the algorithm is demonstrated through phase optimization with fixed magnitude, by targeting optimal RF power delivery. Section II introduces the system structure of the proposed feedback-based WPT system. Section III details the proposed algorithm. Section IV present the simulated and measured results, while Section V concludes the paper.

II. SYSTEM SCENARIO

This section presents the system structure of the conventional feedback-based WPT system. Fig. 1 illustrates a simple configuration. The Transmitter Array (TXA) consists of multiple Transmit Elements (TEs), with the magnitude and

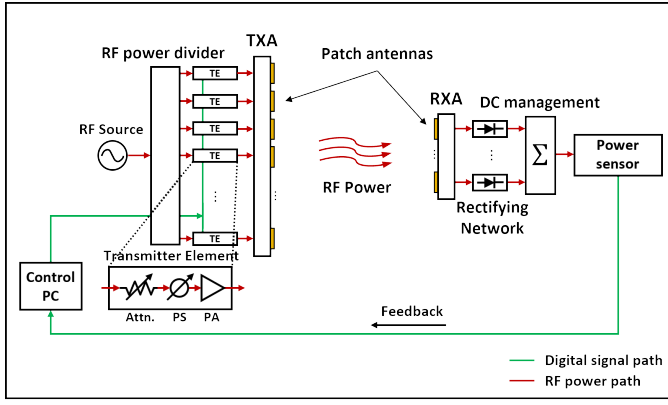


Fig. 1. Conventional feedback-based practical WPT system configuration.

phase of each element controlled individually. The Receiver Array (RXA) includes rectifiers and combining circuits, where a power sensor detects the received power. The detected power at the RXA is fed back to the TXA control PC, where all processing is performed. It is important that the receiver sends only the received power as feedback, which makes it more challenging compared to telecommunications, where the receiver can actively perform beamforming to find better channel conditions.

Based on this conventional configuration, this paper imposes the following constraints for further extensions:

- 1) The system assumes that only the phase is controllable.
- 2) The feedback data is the sum of the total received RF power.²

III. PROPOSED ALGORITHM

This section introduces the proposed algorithm based on phase control. Consider a WPT scenario, as depicted in Fig. 1, with a TXA of size $M \times M$ and an RXA of size $N \times N$. Each Transmit Element (TE) is equipped with an n -bit phase shifter. The objective is to minimize the number of phase trials required to achieve the highest received power.

It is impossible to brute-force all combinations to search for the optimum phase, as the complexity is given by $(2^n)^{M^2}$ cases. However, by dividing the target pattern into orthogonal bases, we can more efficiently explore candidate patterns without wasting lots of time using brute-force methods³. Consequently, the complexity of determining optimal or near-optimal patterns is substantially lowered. Hadamard matrices [9] are widely recognized as one of the most effective methods for defining an orthogonal vector space as a basis. They have been extensively applied in telecommunications [10] as well as in the WPT field [7].

²A more practical scenario that includes rectifiers which feedbacks the DC power will be addressed in future work, considering the additional complexity caused by the non-linearity.

³The Fourier matrix is also closely related to the Hadamard matrix in its application to our case.

A. Hadamard Orthogonal Basis

Hadamard matrix is defined as a square matrix whose entries are either $+1$ or -1 and whose rows are mutually orthogonal. Therefore, for an $n \times n$ Hadamard matrix $\mathbf{H} = (h_{i,j})_{i,j \in \{1, \dots, n\}}$, then the orthogonality can be written as followings:

$$\sum_{i=1}^n h_{k,i} \cdot h_{l,i} = 0, \quad (k \neq l) \quad (1)$$

B. Basis Matrix Sets

Since our TXA is assumed to have a size of $M \times M$, it can be represented as a matrix of the same size. In this case, the number of basis matrices required for the system is M^2 . By selecting each row and column of the Hadamard matrix and multiplying them element-wise, M^2 basis matrices, denoted as $\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_{M^2}$, can be generated. This process can be expressed as follows:

$$\mathbf{B}_k = h_i \otimes h_j^T \quad (2)$$

where h_i and h_j are the i -th and j -th rows, respectively, and \mathbf{B}_k is the k -th basis matrix. Then, the k -th basis transmit pattern \mathbf{T}_k at the TXA⁴ can be written as followings:

$$\mathbf{T}_k = \mathbf{B}_k \cdot \angle \phi_k \cdot \mathbf{1}_{M \times M} \quad (3)$$

where all elements are either $\angle \phi_k$ or $-\angle \phi_k$, as $-\angle \phi_k$ can be rewritten as $\angle(\phi_k + \pi)$.

C. 1D Beam-scanning

However, using all M^2 basis matrices remains complicated. From antenna array theory, the Array Factor (AF) of a 2D array pattern can be derived by the multiplication of two orthogonal 1D vector spaces. Applying this concept, the multiplication of azimuth and elevation 1D vectors can generate the 2D pattern as follows⁵:

$$\mathbf{S}_a = (e^{j\varphi_1^a} \quad e^{j\varphi_2^a} \quad \dots \quad e^{j\varphi_M^a}) \quad (4)$$

$$\mathbf{S}_e^T = (e^{j\varphi_1^e} \quad e^{j\varphi_2^e} \quad \dots \quad e^{j\varphi_M^e}) \quad (5)$$

$$\mathbf{S}_{2D} = \mathbf{S}_e \cdot \mathbf{S}_a = \begin{pmatrix} e^{j\psi_{1,1}} & e^{j\psi_{1,2}} & \dots & e^{j\psi_{1,M}} \\ e^{j\psi_{2,1}} & e^{j\psi_{2,2}} & \dots & e^{j\psi_{2,M}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\psi_{M,1}} & e^{j\psi_{M,2}} & \dots & e^{j\psi_{M,M}} \end{pmatrix} \quad (6)$$

where \mathbf{S}_a and \mathbf{S}_e are the azimuth and elevation 1D matrices, respectively. Multiplying these two matrices produces the 2D pattern shown in (6). Therefore, finding appropriate 1D matrices \mathbf{S}_a and \mathbf{S}_e is crucial. Fig. 2 illustrates the detailed procedure for finding \mathbf{S}_a using the proposed sequential feedback-based method.

⁴Make sure the difference between the \mathbf{B}_k and \mathbf{T}_k , where \mathbf{B}_k is the orthogonal basis itself, while \mathbf{T}_k is the point of the field in exponent expression.

⁵Specifically, The azimuth 1D vector means the row vector, which steers the beam in azimuth direction, while elevation means the column vector, that steers the beam in elevation direction. The words were took by the conventional RADAR systems.

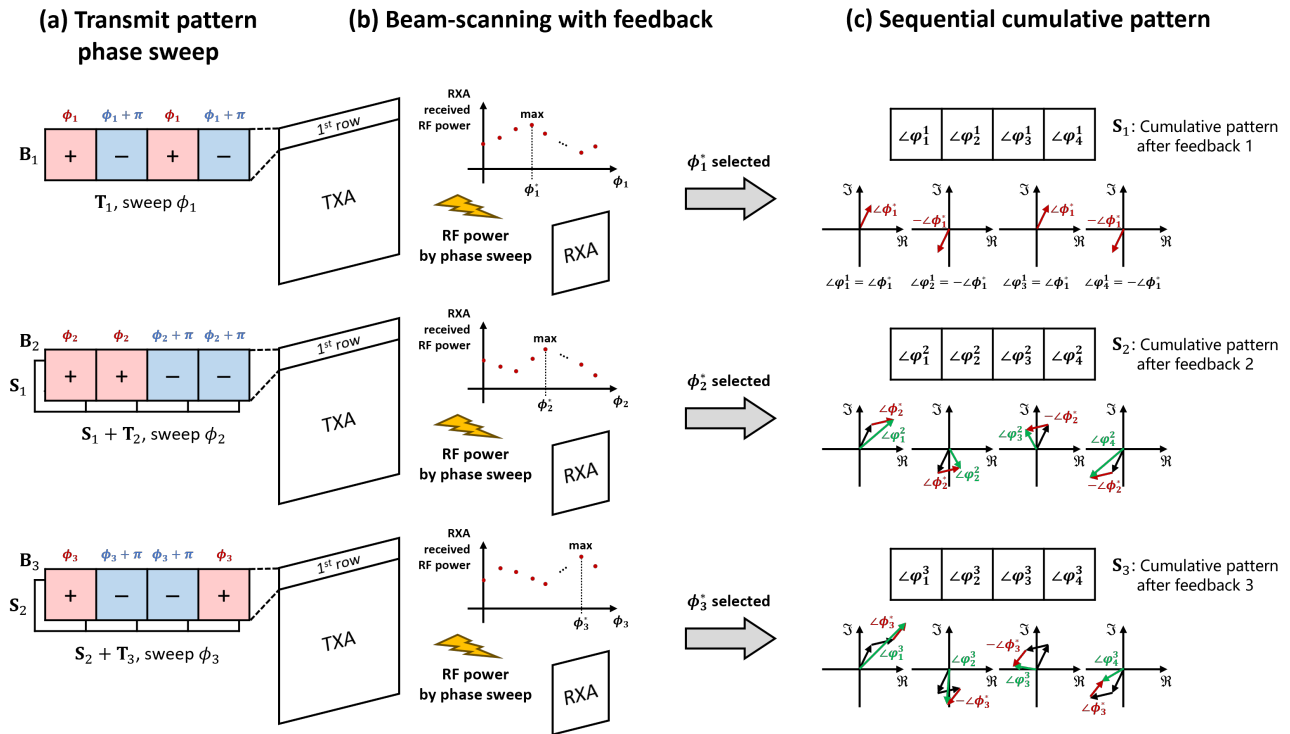


Fig. 2. Proposed beam-scanning algorithm based on sequential phase sweeping with feedback. The figure illustrates the azimuth beam-scanning procedure, where the first row is selected in this case. The elevation procedure follows the same process by selecting either the first or last column.

The algorithm iterates its feedback process by the following three-steps: (a) transmit pattern phase sweep, (b) beam-scanning with feedback, and (c) sequential cumulative pattern generation:

- 1) The first Hadamard basis 1D matrix \mathbf{B}_1 is applied as a mask at the TXA.
- 2) The RXA records the total received RF power as the transmitter sequentially sweeps the phase. Then, the data is fed back to the TXA processor, where the phase corresponding to the maximum power is recorded as ϕ_1^* .
- 3) The cumulative pattern \mathbf{S}_1 records the pattern in its exponential form.
- 4) For the next basis, \mathbf{B}_2 is combined cumulatively with \mathbf{S}_1 .
- 5) Repeat the process until M , where the final pattern \mathbf{S}_a is obtained by normalizing \mathbf{S}_M .

By this procedure, \mathbf{S}_a and \mathbf{S}_e are derived individually, and the final pattern \mathbf{S}_{2D} is calculated using (6). The total complexity of the proposed algorithm is calculated as $2^{n+1} \times M$.

IV. RESULTS

This section presents the simulation and measurement results for the proposed algorithm. The sizes of the TXA and RXA were set to 8×8 and 3×3 , respectively, for both simulation and measurement. The hardware system, operating at 5.64 GHz, includes a patch TXA and RXA with sizes of

16×16 and 7×5^6 , respectively, with 0.6λ element spacing using FR-4 as substrate ($\epsilon_r = 4.4$, $\tan \delta = 0.02$, $h = 1.6$ mm). The phase was controlled for all elements individually by 4-bit phase shifter with a resolution of 22.5° [11], [12]. For the simplest scenario setup, the distance between the TXA and RXA was set to 0.5 m, with both aligned directly facing each other along the z -axis.

A. Simulation Results

The proposed algorithm processes azimuth (1st row) and elevation (1st column) beam-scanning individually and combines them into a 2D pattern using (6). Since the TXA has a size of 8×8 , eight basis matrices are generated. However, as the first row of the Hadamard matrix always consists entirely of ones, it cannot produce relative phase differences. Consequently, only 7 basis matrices are required. The TXA pattern that yields the highest received power is selected as the final pattern for each azimuth and elevation beam-scanning.

Fig. 3 shows the simulated results obtained by sweeping the phase and iterating through Hadamard masks while measuring the total RF power at the receiver. Since this is a simulation result, the time required for the full phase sweep procedure was under 1 second due to the fast computational speed. The total time consumption increased linearly throughout the process, due to the simplicity and low complexity of the algorithm.

⁶Although the hardware system supports a size of 16×16 , this paper uses a size of 8×8 for simplicity. Future work will consider the full-size configuration under various scenarios.

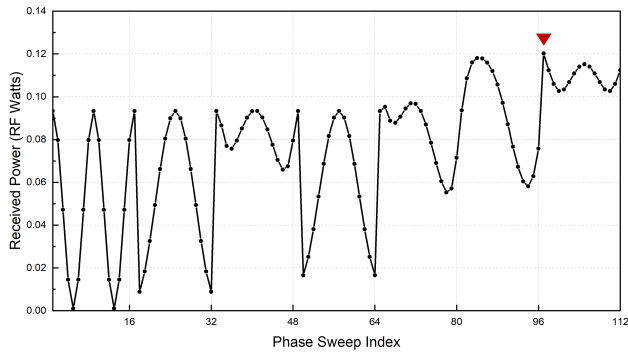


Fig. 3. The result of the azimuth beam-scanning.

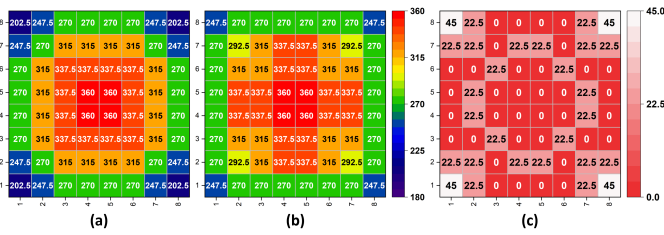


Fig. 4. Heatmap of the quantized and normalized transmitted phase patterns (in degrees). (a) Convex optimization, (b) proposed beam-scanning, and (c) absolute difference between (a) and (b).

In practical scenarios, the total duration may depend on the speed of the feedback communication module itself, such as Bluetooth, Zigbee, etc. During the entire beam-scanning process, the magnitude was fixed at 24 dBm⁷. The red triangle symbol marks the point where the maximum power was received, representing the optimal pattern selected. The graph illustrates the received power gradually increasing during the phase sweep, requiring only 112 attempts for a full sweep^{8 9}. Since the RXA was located at the center facing the TXA, the elevation beam-scanning pattern was identical to the azimuth pattern.

The 2D pattern was calculated using (6), and the total received power was also computed. Fig. 4 shows the phase pattern (in degrees) of the transmitted power, quantized to 22.5° considering the 4-bit phase shifter. Fig. 4 (a) presents the optimal pattern (which optimizes the RF-to-RF efficiency) calculated using convex optimization (CVX) [13]¹⁰, while (b) shows the proposed pattern, and (c) illustrates the absolute difference between the two. The results indicate that the phase distribution of the proposed beam-scanning method closely approximates the optimal pattern.

⁷The rows from 2 to 8 were set to “off”

⁸7 basis matrices \times 2⁴ phase sweeps = 112 attempts.

⁹This graph is quite simple, but considering the symmetrical location scenario for this paper, it is reasonable that the power doesn't increase dramatically. However, for the future expansions, the graph is to be expected to be more complex when the receiver location is more misaligned.

¹⁰This optimized pattern includes both magnitude and phase, so comparing directly with the proposed pattern might cause a little error, but the tapered magnitude pattern is actually observed as nearly as an uniform distribution near 24 dBm.

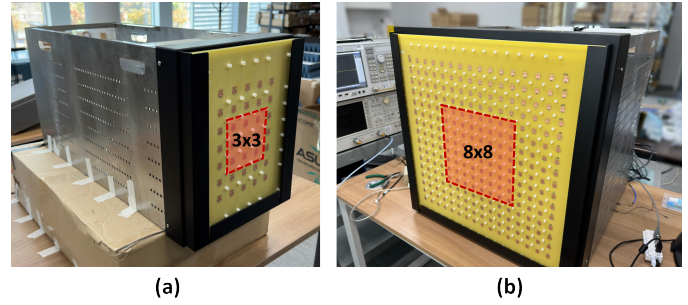


Fig. 5. TXA and RXA in the measurement setup. The center 8×8 and 3×3 arrays were selected for the TXA and RXA, respectively.

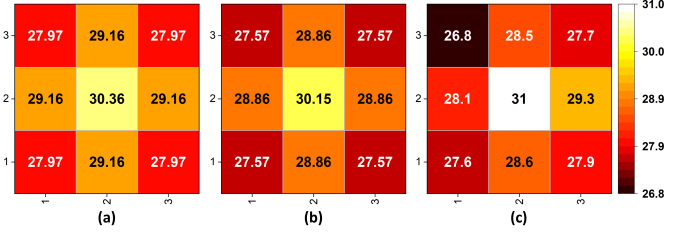


Fig. 6. Heatmap of the received power (in dBm) for different transmit patterns in RF watts. (a) Convex optimization (simulated), (b) Proposed beam-scanning (simulated), and (c) Proposed beam-scanning (measured).

B. Measurement Results

It is not feasible to directly feedback the total received RF power; therefore, this paper validates the pattern of the proposed beam-scanning method instead¹¹. Fig. 6 presents the received RF power results, where (a) and (b) show the simulated values, and (c) shows the measured values by Spectrum Analyzer.

The total simulated RF power using the CVX pattern and the proposed pattern was calculated as 6.89 W and 6.40 W, respectively, while the measured power for the proposed method was 6.44 W. The measured result for the proposed method closely matches the simulated value and is also very close to the optimal power. This demonstrates that the proposed beam-scanning method effectively approximates the optimal pattern.

V. CONCLUSION AND FUTURE EXTENSIONS

This paper proposes a low-complexity phase optimization algorithm for WPT systems using the orthogonal basis provided by Hadamard matrices. By dividing the overall search space into orthonormal bases, the algorithm significantly reduces the computational complexity involved in determining optimal phase configurations. The system proposed the novelties of sequential cumulative feedback to iteratively reach the optimal phase distribution, and further reduces complexity by decomposing the 2D pattern into 1D patterns, focusing solely on maximizing received power without adjusting magnitude. The results, supported by both simulated and measured data,

¹¹The complete active feedback process will be proposed in future work by using merged DC power feedback.

demonstrate the feasibility and effectiveness of the proposed method.

Future work may explore extensions to arbitrary receiver locations, which could require more detailed development. In particular, simply multiplying the two 1D patterns to construct a 2D pattern as in (6) is not accurate in near-field applications, and additional processes may need to be included to reach higher efficiency. Finally, extensions to nonlinear systems, such as incorporating rectifiers with merged DC power feedback, could enhance its practicality.

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